with a small detector providing sensitivity to deposition energies of 1–20 keV is obtained. A significant signal is still expected when probing incident cosmions with velocities near the galactic escape velocity of 2×10^{-3} c, in which case the recoil energies will be 4–80 keV. These event rates also apply to superconducting colloid detectors[6] which could also detect cosmions.

Cosmions present a far easier target for bolometric detection than do solar neutrinos. The Press-Spergel hypothesis can be tested decisively. Should the existence of cosmions be demonstrated, bolometric measurements (in Si, B and possibly other materials) can yield their mass, local density, velocity distribution and the magnitude and spin-dependence of cosmion-nucleon cross-sections.

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The real value of Mercury's perihelion advance

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The perihelion advance of the orbit of Mercury has long been one of the observational cornerstones of general relativity. After the effects of the gravitational perturbations of the other planets have been accounted for, the remaining advance fits accurately to predictions according to the theory of general relativity, that is, 43 arc s per 100 yr. In fact, radar determinations of Mercury's motion since the mid-1960s have produced agreement with general relativity at the level of 0.5%, or to ±0.2 arc s per 100 yr. We have recently been puzzled by an apparent uncertainty in the value of the theoretical prediction for the advance within general relativity, as cited in various sources. The origin of this discrepancy is a 1947 article by Clemence[1], who used unconventional values for the astronomical unit (A) and the speed of light (c) in his calculation of the predicted advance. Since that time, virtually everyone has followed suit. Although the current value of the prediction, using the best accepted values for the astronomical constants and for the orbital elements of Mercury, is 42.98 arc s per 100 yr, there is a preponderance of citations over the past 25 years of Clemence's value 43.03 arc s per 100 yr. Here we derive the accurate value and uncover the source of Clemence's value.

According to general relativity, the predicted perihelion precession rate ($\dot{\omega}$) is given by[2]

$$\dot{\omega}=\frac{6\pi GM_\odot}{Pa(1-e^2)c^2} \qquad (1)$$

where G is the newtonian gravitational constant, $M_\odot$ is the mass of the Sun and a, e and P are the semi-major axis, eccentricity and orbital period of Mercury, respectively, whose mass has been neglected with respect to $M_\odot$. However, to obtain a numerical value for $\dot{\omega}$, note that $M_\odot$ cannot be measured directly. Instead the product $GM_\odot$ is determined through measurements of the orbital periods (or mean motions) and the semi-major axes of planetary orbits, and the use of Kepler's third law. In relativistic language, $GM_\odot$ is said to be the 'Kepler-measured' mass, as distinct from the total rest mass of all the particles contained in the Sun. This Kepler-measured mass is given by

$$GM_\odot=n^2a^3 \qquad (2)$$

where $n=2\pi/P$ is the mean motion of a body of negligible mass in an orbit of semi-major axis a. It is conventional to define the gaussian gravitational constant k, given by

$$k^2\equiv\bar{n}^2\bar{a}^3 \qquad (3)$$

where $\bar{n}$ is the mean motion in radians per mean solar day of a body of negligible mass with respect to $M_\odot$ (unit mass) and $\bar{a}$ is its semi-major axis in AU. By international agreement, k has been given the defining value of 0.01720209895 since 1938. This is the value first assigned to it by Gauss on the basis of values available at that time for the Earth's orbital period and for the Earth–Sun mass ratio. This value serves to define A as the radius of a circular orbit around the Sun on which a body of negligible mass would complete one revolution in $2\pi/k=365.2568983$ mean solar days. The solar mass is then related to A by

$$GM_\odot=k^2A^3/D^2 \qquad (4)$$

where A is in metres, D = mean solar day = 86,400 s and $GM_\odot$ is expressed in $\mathrm{m^3\,s^{-2}}$. The semi-major axes of planetary orbits are usually measured in units of A, so if we define $\bar{a}=a/A$, we obtain for $\dot{\omega}$

$$\dot{\omega}=\frac{6\pi k^2}{PD^2\bar{a}(1-e^2)}\left[\frac{A^2}{c^2}\right] \qquad (5)$$

Now, k and D are defined constants, while for Mercury, the mean orbital elements $\bar{a}$, e and P are known accurately and have maintained constant values to better than 1 part in 10^5 for at least the past 80 years. Those values are $\bar{a}=0.387099$, $e=0.205630$ and $P=0.24085$ yr. From the International Astronomical Union (IAU) 1976 System[3] (Table 1), the current adopted values for A and c are

$$A_{1976}=1.4959787\times10^{11}\ \mathrm{m} \qquad (6)$$

$$c_{1976}=299{,}792{,}458\ \mathrm{m\,s^{-1}} \qquad (7)$$

In terms of these values, we obtain

$$\dot{\omega}=42.98(A/A_{1976})^2\,(c/c_{1976})^{-2}\ \text{arc s per 100 yr} \qquad (8)$$

However, A and c have not always had their current values. Before 1964, the value of the astronomical unit was a *derived* quantity, obtained from the solar horizontal parallax (the angle subtended by the Earth's equatorial radius at 1 AU), which was treated as a primary constant, and had the adopted value of 8.8 arc s, and from the radius of the Earth, also treated as a primary constant with the value 6.378388×10^6 m (see Table 1). The derived value of A was 1.495042×10^{11} m. But by the mid-1960s, radar ranging to the inner planets produced direct and more accurate determinations of distance (through measurements of light travel time) and of A, and it was found that the previous value was too small by ~90,000 km. By 1964, IAU had adopted the radar value as a primary constant, and relegated the solar parallax to the status of a derived constant[4]. Minor refinements in the value of A were made in the 1976 IAU system. Similarly, the value of c that was used before the mid-1960s was 299,860,000 $\mathrm{m\,s^{-1}}$, a value dating from Michelson's measurements of the late nineteenth century. It was replaced by the lower value 299,792,500 $\mathrm{m\,s^{-1}}$, based on modern measurements.

With the older values of A and c, the predicted perihelion advance before 1964 would have been 42.91 arc s per 100 yr. With the current values, the prediction is 42.98 arc s per 100 yr. Where then did the value 43.03 come from?

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This value originates from an article[1] by Clemence which reviews the observations of the overall perihelion shift, the subtraction of the general precession of the geocentric coordinate system and the removal of the contribution from planetary perturbations. However, Clemence's analyses adopted what were at the time unconventional values of A and c. Instead of the accepted value of 8.8 arc s for the solar parallax, he chose a value of 8.790 arc s, which, combined with the conventional Earth radius, gave a value for A of 1.4967429×10^{11} m. This value is slightly closer to the current value, although ~0.05% too high, than was the conventional 1947 value, which was about 0.06% too low. Furthermore, for c he also chose a non-conventional value which he took from a review by Dorsey[5] of measurements of c. This value, 299,773,000 m s^{-1}, was lower than the Michelson value, although quite close to the current value. The other constants for Mercury's orbit were the standard values. His result for the general relativistic prediction was 43.03 arc-s per 100 yr.

Clemence's value was generally accepted for 35 years. In many textbooks and review articles on general relativity, authors quote the rounded-off values 43 or 43.0 arc s per 100 yr, both of which are compatible with either Clemence's or the present-day value. However, of those authors who quote four-figure accuracy, most agree with Clemence. A survey of the literature shows 18 publications since 1960 containing Clemence's value, but only 4 with the modern value. The distribution of these citations is plotted as a function of year in Fig. 1.

Two cases in point are the classic textbooks on gravitation by Misner, Thorne and Wheeler[2] and by Weinberg[6]. Both texts list in their appendices values for A and c which are consistent with the 1964 IAU System, yet both fall into step behind the Clemence perihelion prediction (p. 1113 in ref. 2; p. 198 in ref. 6). Clemence's[1] value is also maintained in other major textbooks[7-12]. We have been unable to determine why Clemence's value was so widely used.

Figure 1 also shows the value 42.95 arc s per 100 yr quoted by Richard[13] and by Will[14-16]. This value is anomalous and appears to be the product of rounding off one of the ingredients of the prediction (such as the orbital period) too severely before inserting into the formula.

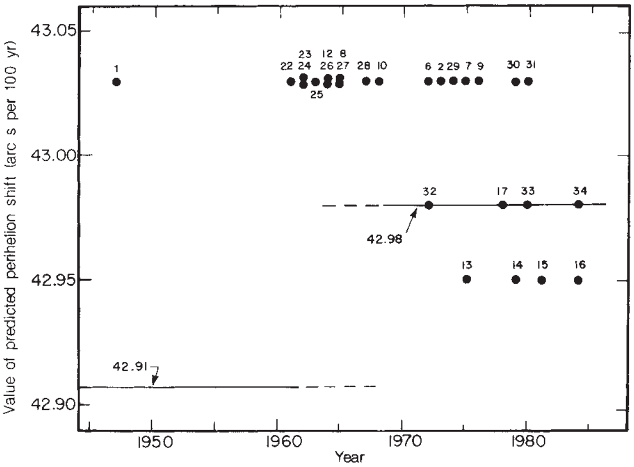

Fig. 1 Quoted values of the general relativistic prediction for Mercury's perihelion advance 1947-85. Horizontal lines are the values dictated by currently accepted values for A and c. Numbers above circles are references.

Although of theoretical interest, the difference between these quoted predictions for Mercury's perihelion advance has no observational consequences, for the following reason. In the current method of testing gravitational theory using the dynamics of Mercury, the actual value of the predicted advance is irrelevant. Instead, the equations of motion used to determine the ephemerides of Mercury and the inner planets automatically include relativistic post-newtonian terms whose time-averaged consequence is the perihelion shift. The size or effect of these terms in the equations of motion is modulated by a set of dimensionless parameters, 'parametrized post-newtonian (PPN) parameters', whose values depend on the theory of gravitation being used (they have specific fixed values in general relativity)[15]. These parameters, termed, for example, γ, β, α_1, become part of a multi-parameter least-squares fit procedure in which radar data and other appropriate measurements are used to improve the parameters in the least-squares sense[17,18]. Other parameters include the usual ones, for example, initial conditions for the planets, station locations, A and the Earth-Moon mass ratio. The outcome of such experiments is an improved set of values for all the parameters. The values of the PPN parameters can then be compared with their predicted values in various theories. Thus, although it is possible that the accuracy of the determinations of the PPN parameters by this method may eventually reach an accuracy corresponding to the 0.05 arc s per 100 yr difference between the Clemence value for $\dot{\omega}$ and the true value, the actual value for the net perihelion precession never appears in the analysis. Current observations combining Mercury radar data with Viking Mars data are nearing this level of accuracy[18], and a possible future Mercury orbiter may do even better[19].

The interpretation of the perihelion advance is further complicated by the contribution of a newtonian quadrupole gravitational field generated by a solar oblateness. The possible contributions to $\dot{\omega}$ depend on the size of the solar mass quadrupole moment, as inferred from various observations or from theoretical models of the Sun. The contributions range in size from 3 arc s per 100 yr, inferred from the original Dicke-Goldenberg 1966 visual solar oblateness measurements, to 1 arc s per 100 yr, from more recent 1983 visual observations[20], to 0.6 arc s per 100 yr, inferred from some solar oscillation measurements[21], to 0.1 arc s per 100 yr, inferred from Mercury/Viking Mars data[18], to 0.01 arc s per 100 yr, inferred from standard, uniformly rotating theoretical solar models.

Nevertheless, the real general relativistic perihelion shift of Mercury can be stated unequivocally: it is 42.98 arc s per 100 yr.

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Table 1 Astronomical constants and Mercury's perihelion

Constant	1947 (ref. 35)	1947 (ref. 1)	1960 (ref. 35)	1964 IAU system[4]	1976 IAU system[3]
AU A (10^{11} m)	(1.495042)	(1.4967429)	(1.495042)	1.49600	1.4959787
Earth radius (m)	6,378,388	6,378,388	6,378,388	6,378,160	6,378,140
Parallax (arc s)	8.8	8.790	8.8	(8.794)	(8.794148)
c (m s^{-1})	299,860,000	299,773,000	299,860,000	299,792,500	299,792,458
$\dot{\omega}$ (arc s per 100 yr)	42.91	43.03	42.91	42.98	42.98

Values in parentheses are derived constants.

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Extragalactic nature of G227.1 + 1.0

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Green and Gull[1] have presented evidence that G227.1 + 1.0, one of the small-diameter radio sources in the Molonglo survey of Clark and Crawford[2], is a Crab-like supernova remnant. If this is the case, the object is an extremely important one, as Green and Gull emphasize[1]. Such remnants are rare and are valuable as probes of the particle acceleration by pulsars. The very small diameter (~ 1′) of G227.1 + 1.0 implies that it is one of the youngest of these remnants, perhaps only a few hundred years old. It could then represent the first known example of a Phase I Crab-like remnant[3]; that is, one for which the age is less than the slowing timescale of the underlying pulsar. Such a remnant would be uniquely important in helping to establish by observation an evolutionary picture of these objects (see ref. 4 for a theoretical discussion). Furthermore, the direction of G227.1 + 1.0—towards the galactic anti-centre—makes it a good candidate for optical study, as the usual problems of crowding and high extinction in the plane should be reduced. Here we report the discovery of a bright ($V = 17.4$), diffuse optical object within a few arc seconds of the centre of the radio source, but we present spectroscopic evidence that it is a luminous external galaxy at $z = 0.073$. We argue, furthermore, that the original radio emission represents not a supernova remnant, but a radio halo surrounding this galaxy, similar to that around M87.

Figure 1 is a V image of the G227.1 + 1.0 field taken with the prime-focus charge coupled detector (CCD) at the 4-m telescope of the Cerro Tololo Inter-American Observatory (CTIO) on 19 February 1985. The diffuse nature of the central object in the field is readily apparent. The centre of the radio emission (roughly 1′ by 2′ in extent) is not particularly well defined (there is no central point source), but lies roughly 0.1′ west of the centre of the optical emission. The magnitudes of the diffuse object, uncorrected for reddening, were determined from this and other CCD exposures to be $U = 20.1$, $B = 18.9$, $V = 17.4$ and $I = 15.4$, all accurate to 0.1 mag.

Figure 2 shows a spectrum of the optical object obtained with the Lick 3-m CCD spectrograph on 20 March 1985. The spectrum is similar to that of M87 (refs 5, 6) and G127.1 + 0.5, except that the emission is weaker (here only [N II] at λ6,584 is clearly seen in emission), which suggests that the object is a giant elliptical galaxy. The redshift obtained from the emission line and the three absorption features labelled in the figure is $z = 0.073$. For such a galaxy, this redshift then implies $(B - V)_z = 1.2$ (refs 7, 8) so that in terms of magnitude, $V_0 = 16.5$ and $M_V = -21.7$ (for $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$), ~1.5 magnitudes fainter than M87. We note that the extinction along this line-of-sight through the galactic plane is remarkably low ($A_v \sim 0.9$) for $b = 1.0°$.

The analogy to M87 is strengthened if we identify the radio emission reported by Green and Gull[1] with this external galaxy. M87 shows radio emission on a variety of angular scales[9,10], but the most relevant here is the halo, 12′ by 16′ in extent, with a mean surface brightness at 408 MHz of 3.2×10^{-19} $\text{W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}$ (ref. 11). By comparison, the 1-GHz surface brightness of G227.1 + 1.0 reported by Green and Gull[1] extrapolates to almost the identical value at 408 MHz: $3.4 \times 10^{-19} \text{ W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}$, and this object is about a factor of 2 larger than M87 in spatial extent. This close analogy suggests strongly that the radio source G227.1 + 1.0 is associated with the optical galaxy, and does not represent a young galactic supernova remnant. The failure to detect a compact nuclear source in this object is perhaps not surprising in view of the

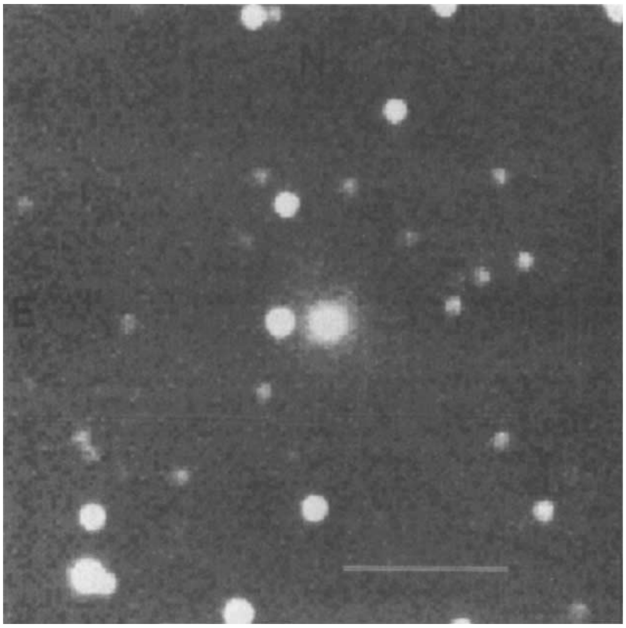

Fig. 1 V image of field of G227.1 + 1.0 taken with the prime-focus CCD at the 4-m telescope of CTIO. The extended nature of the central object, which we identify as a giant elliptical galaxy, is readily apparent. Scale bar represents 20″ in length. Exposure time was 20 seconds.